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ABSTRACT

Equations of motion appropriate to the conditions existing in the ionosphere are discussed with a view to examine the condition for ambipolar diffusion ($\vec{v}_e = \vec{v}_i$). It is shown that for quasi-equilibrium and isothermal conditions the required condition for ambipolar diffusion is given by curl $\vec{v} \times \vec{B} = 0$. It is further shown that the assumption of ambipolar diffusion along the field lines leads to the trivial situation of hydrostatic distribution of electron density independent of latitude.

These results are not in agreement with the generally accepted view that diffusion of the plasma along the direction of the magnetic field can account for many geophysical phenomena in the ionosphere. This disagreement is attributed to the fact that the assumption of field-aligned plasma diffusion puts a constraint on grad n_e which has not been taken into account by the previous workers. It is pointed out in the present paper that the solution of $\vec{v}_e$ and $\vec{v}_i$ in terms of the particle densities and temperatures are not possible without the knowledge of the electric field. The theoretical determination of the latter appears to be extremely complicated and it seems desirable to measure it experimentally.

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Introduction

It now is generally accepted that diffusion plays an important role in controlling the distribution of ionization in the F-region of the ionosphere. The theory of diffusion appropriate to the conditions existing in the ionosphere was first proposed by Ferraro (1945) who treated the electron-ion gas as a single constituent and showed that the coefficient of diffusion of this gas is reduced by the ratio of $1:\sin^2 I$ in the presence of the magnetic field, I being the inclination (dip) of the earth's magnetic field. According to Ferraro, the vertical component of the velocity of diffusion v_z, for an isothermal condition is given by

$$v_z = - D \sin^2 I \left[\frac{1}{n_e} \frac{\partial n_e}{\partial z} + \frac{1}{H_1}\right] \tag{1}$$

where D is the coefficient of diffusion, n_e the electron-density H_1, the scale height of electron-ion gas, and z the altitude.

Based on equation (1) the diffusive equilibrium-distribution at places other than the magnetic equator is given by

$$n_e = n_{eo} e^{-\left(\frac{z-z_o}{H_1}\right)} \tag{2}$$

where n_{eo} is the electron-density at height z_o.

Equation (2) is in general accordance with the experimentally observed distribution well above the F_2-peak both at midlatitudes and above the equator even though according to equation (1), the vertical diffusion is inhibited at the geomagnetic equator.

Several modifications have been proposed to the original theory of Ferraro to take into account the temperature gradient and the effect of horizontal gradients but it has always been assumed or implied that the diffusion is essentially ambipolar ($\vec{v}_e = \vec{v}_i$). Johnson and Hulburt (1950), who treated the problem of plasma diffusion in the ionosphere in great detail showed that the electron-ion gas may diffuse together as a single constituent in the absence of the magnetic field, with a coefficient of diffusion and scale height twice that of the positive ions. In the presence of the magnetic field, however, diffusion is not ambipolar and is affected by the force exerted by the magnetic field on the electrical currents.

In view of the generally-accepted conclusion that diffusion plays a significant role in controlling the charged particle distribution in the upper ionosphere, it is important to examine if the condition for ambipolar diffusion actually exists in the ionosphere where the effect of the magnetic field is significant. The purpose of this paper is to investigate this problem in detail and to specify the conditions which must be satisfied before the assumption of ambipolar diffusion can justifiably be used in the ionosphere.

Equation of Motion: In a multiple-component gas under the action of external forces, the equation of motion obeyed by each constituent may be written in the form proposed by Johnson (1951) and Schlüter (1951).

$$\frac{\partial \vec{v}_s}{\partial t} + \vec{v}_s \cdot \frac{\partial \vec{v}_s}{\partial \vec{r}} = - \frac{1}{\rho_s} \frac{\partial}{\partial \vec{r}} \cdot \vec{\vec{p}}_s + \vec{F}_s + \sum_{\ell} \frac{m_\ell \nu_{s\ell}}{m_s + m_\ell} (\vec{v}_\ell - \vec{v}_s) \qquad (3)$$

where the suffixes s and ℓ stand for the type of the particles and the various terms in equation (3) may be defined as follows:

$\vec{v}_s$ = macroscopic velocity of the sth constituent

ρ_s = density

$\overleftrightarrow{p}_s$ = pressure tensor; in general its ij-th element is given by the following equation (Lamb 1932)

$$(p_s)_{ij} = (p_s\delta_{ij} + \frac{2}{3}\rho_s\eta_s \frac{\partial v_{sk}}{\partial x_k}\delta_{ij}) - \rho_s\eta_s(\frac{\partial v_{is}}{\partial x_j} + \frac{\partial v_{sj}}{\partial x_i}) \qquad (4)$$

where η_s stands for the coefficient of kinetic viscosity.

$$\vec{F}_s = \frac{e_s}{m_s}(\vec{E} + \vec{v}_s \times \vec{B}) + 2\vec{v}_s \times \vec{\omega} - \nabla\Omega_{tidal} + \vec{g} \qquad (5)$$

where $\vec{E}$, $\vec{B}$, e_s are, respectively, the electric field, the magnetic field and the charge, all expressed in MKS units, Ω_{tidal} is the tidal force due to the sun and the moon and $\vec{g}$ is the acceleration due to gravity. The terms $2\vec{v}_s \times \vec{\omega}$, known as the Coriolis acceleration appears because of the rotation of the terrestrial coordinate system with the angular frequency $\vec{\omega}$.

The last term in equation (3) represents the drag term. The symbols m_s and m_ℓ are the masses of sth and ℓth kind of particles and $\nu_{s\ell}$ is the collision frequency of sth kind of particles with ℓth. The summation with respect to ℓ is extended to cover all possible collision partners including $s = \ell$.

In discussing the problem of diffusion in the ionosphere we shall assume only three types of particles: electrons, ions and neutral atoms. Further, to simplify our discussion we shall ignore the effect of viscosity, Coriolis-and tidal forces and consider the equation corresponding to quasi-equilibrium conditions. The neglect of viscosity removes the off-diagonal terms from the pressure term. Further, p may be treated as a scalar given by the equation of state

$$p_s = n_s k T_s \qquad (6)$$

where k is the Boltzmann constant and T_s the kinetic gas temperature.

In a collision-dominated plasma the assumption of isotropicity of pressure is justified (Spitzer; 1962). Finally, we shall neglect the quadratic terms in $\vec{v}_s$ and its derivative, thereby, linearising all the equations. The approximation made so far are just the ones usually made in the study of the diffusion problem in the ionosphere. The equations of motion for neutrals, electrons, ions may now be written in the following form

$$n_n n_e\, \alpha_{en}(\vec{v}_n-\vec{v}_e) + n_n n_i\, \alpha_{in}(\vec{v}_n-\vec{v}_i) = -\nabla p_n + \rho_n \vec{g} \qquad (7)$$

$$n_e n_i\, \alpha_{ei}\, (\vec{v}_e-\vec{v}_i) + n_n n_e\, \alpha_{en}\, (\vec{v}_e-\vec{v}_n) = -\nabla p_e + \rho_e\, \vec{g} - e n_e\, (\vec{E} + \vec{v}_e \times \vec{B}) \qquad (8)$$

$$n_e n_i\, \alpha_{ei}\, (\vec{v}_i-\vec{v}_e) + n_n n_i\, \alpha_{in}\, (\vec{v}_i-\vec{v}_n) = -\nabla p_i + \rho_i\, \vec{g} + n_i e\, (\vec{E} + \vec{v}_i \times \vec{B}) \qquad (9)$$

where

$$\alpha_{ei} = \frac{m_e\, m_i\, \nu_{ei}}{(m_e+m_i)\, n_i} \qquad (10A)$$

$$\alpha_{en} = \frac{m_e\, m_n\, \nu_{en}}{(m_e+m_n)\, n_n} \qquad (10B)$$

$$\alpha_{in} = \frac{m_i\, m_n\, \nu_{in}}{(m_i+m_n)\, n_n} \qquad (10C)$$

The suffixes e, i, n in equation 7-10 stand for electron, ion and neutral molecule. For an isothermal condition, α_{ei}, etc. may be treated as constants.

The electric field $\vec{E}$, in general, is the sum of external and internal fields. Equations 7-9 should be supplemented by Maxwell's equations and equations of continuity. We may then write

$$\nabla . \vec{E} = (n_i - n_e)\frac{e}{\varepsilon_o} \tag{11}$$

where ε_o is the free space permittivity

$$\nabla \times \vec{E} = 0 \tag{12}$$

$$\nabla . n_s \vec{v}_s = R_s \tag{13}$$

where R_s refers to the net volumetric rate of creation of sth kind of particles.

We may derive from equations 7-9, the following set of equations which will be useful in the subsequent discussion.

$$\vec{v}_n = \frac{n_e \alpha_{en} \vec{v}_e + n_i \alpha_{in} \vec{v}_i}{n_e \alpha_{en} + n_i \alpha_{in}} + \frac{\vec{A}_n}{n_n (n_e \alpha_{en} + n_i \alpha_{in})} \tag{14}$$

$$\vec{A}_e + \vec{A}_i + \vec{A}_n + e\ (n_i - n_e)\vec{E} + e\ (n_i \vec{v}_i - n_e \vec{v}_e) \times \vec{B} = 0 \tag{15}$$

where $$\vec{A}_{e,i,n} = - \nabla p_{e,i,n} + \rho_{e,i,n} \vec{g} \tag{16}$$

$$L(n_e \alpha_{en} + n_i \alpha_{in})(\vec{v}_e - \vec{v}_i) + e\ (\alpha_{en} \vec{v}_i + \alpha_{in} \vec{v}_e) \times \vec{B}$$
$$+ e(\alpha_{en} + \alpha_{in})\ \vec{E} = \alpha_{in} \frac{\vec{A}_e}{n_e} - \alpha_{en} \frac{\vec{A}_i}{n_i} \tag{17}$$

where $$L = \left(\alpha_{ei} + \frac{n_n \alpha_{en} \alpha_{in}}{n_e \alpha_{en} + n_i \alpha_{in}}\right) \tag{18}$$

Ambipolar Diffusion

In the following we shall investigate the condition under which the electron-ion plasma may diffuse together with a common velocity $\vec{v}$ (usually called ambipolar diffusion or plasma diffusion) such that

$$\vec{v}_e = \vec{v}_i = \vec{v} \tag{19}$$

substituting equation (19) in equation (17), we obtain

$$e(E + \vec{v} \times \vec{B}) = \frac{\alpha_{in}}{\alpha_{en}+\alpha_{in}} \frac{\vec{A}_e}{n_e} - \frac{\alpha_{en}}{\alpha_{en}+\alpha_{in}} \frac{\vec{A}_i}{n_i} \tag{20}$$

It is clear that equation (20) must be satisfied if equation (19) is valid. The required condition therefore is given by

$$e \nabla\times \vec{v} \times \vec{B} = \nabla\times \left[\frac{\alpha_{in}\vec{A}_e/n_e}{\alpha_{en}+\alpha_{in}} - \frac{\alpha_{en}\vec{A}_i/n_i}{\alpha_{en}+\alpha_{in}}\right] = 0 \tag{21}$$

since $$\nabla\times \vec{E} = 0$$

It is easy to verify that the R.H.S. of equation (21) is zero for an isothermal condition. Thus, the assumption of $\vec{v}_e = \vec{v}_i$ leads to the following condition.

$$\nabla\times \vec{v} \times \vec{B} = 0 \tag{22}$$

This condition is always fulfilled in the absence of a magnetic field or when the motion is along the field lines. The last condition is generally assumed to be valid in the F-region and in the following we shall examine this case in detail.

Diffusion Along Field Lines

From equation (14) and (19)

$$\vec{v} - \vec{v}_n = \frac{-\vec{A}_n}{(n_e\alpha_{en}+n_i\alpha_{in})n_n} \tag{23}$$

substituting equation (15) and equation (20) in equation (23) and assuming

$$m_e<<m_i \text{ and } \frac{n_i-n_e}{n_{e,i}} <<1, \text{ we get}$$

$$\vec{v} - \vec{v}_n \simeq - \frac{2kT}{n_n(\alpha_{en}+\alpha_{in})}\left[\frac{1}{n_e}\nabla n_e - \frac{m_i\vec{g}}{2kT}\right] \tag{24}$$

In deriving equation (24), it is assumed that $T_e = T_i = T$. It is evident that equation (24) does not explicitly depend on the magnetic field. In the following, we assume for simplicity that $\vec{v}_n<<\vec{v}$. This assumption may not be justifiable in general. However, it can be easily verified that equation (22) still holds even in this case. Equations (22) and (24) thus lead to the following equation

$$e\nabla\times \vec{v} \times \vec{B} = \nabla\times \left[\frac{-2kT}{n_n(\alpha_{en}+\alpha_{in})} \left(\frac{1}{n_e}\nabla n_e - \frac{m_i\vec{g}}{2kT}\right) \times \vec{B}\right]=0 \tag{25}$$

Equation (25) specifies the condition for ambipolar diffusion and must be solved to determine the required distribution. In the case of field-aligned motion, i.e., when the plasma is diffusing along the field lines, equation (25) is clearly satisfied. We shall examine this case in the following.

In a spherical polar coordinate system coincident with the center of the earth, we may write

$$v_r = \frac{-2kT}{n_n(\alpha_{en}+\alpha_{in})}\left[\frac{1}{n_e}\frac{\partial n_e}{\partial r} + \frac{m_i g}{2kT}\right]$$

$$v_\theta = \frac{-2kT}{n_n(\alpha_{en}+\alpha_{in})}\frac{1}{n_e}\frac{\partial n_e}{r\partial\theta} \tag{26}$$

$$v_\varphi = \frac{-2kT}{n_n(\alpha_{en}+\alpha_{in})}\frac{1}{n_e}\frac{1}{r\sin\theta}\frac{\partial n_e}{\partial\varphi}$$

where r is measured positive outward and θ and φ denote the geomagnetic colatitude and longitude. Further we may write

$$\vec{B} = -B\,(\vec{i}_r \sin I + \vec{i}_\theta \cos I) \tag{27}$$

where $\vec{i}_r$ and $\vec{i}_\theta$ are unit vectors along r and θ directions and I is the magnetic dip angle reckoned positive when the north seeking pole of the needle points downward. The field aligned plasma diffusion case using equations (26) and (27) yields the following equation

$$\frac{1}{n_e}\frac{\partial n_e}{\partial r} + \frac{1}{2H} = \frac{1}{n_e}\frac{1}{r}\frac{\partial n_e}{\partial\theta}\tan I \tag{28}$$

where

$$H = \frac{kT}{m_i g}$$

Equation (28) is a partial differential equation and we shall make use of it in solving the equation of continuity. Equation (13), putting $R_e = 0$, may be written

$$\nabla \cdot n_e \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 n_e v_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial\theta} (n_e v_\theta \sin\theta)$$

$$+ \frac{1}{r \sin\theta} \frac{\partial}{\partial\varphi} (n_\varphi v_\varphi) = 0 \tag{29}$$

Again from equations (15, 19 and 20) assuming $m_i \simeq m_n$ and neglecting n_e or n_i as compared to n_n we may write

$$\frac{1}{n_n} \frac{\partial n_n}{\partial r} \simeq - \frac{m_n g}{kT}$$

$$n_n \simeq n_{no} e^{-\left(\frac{r-r_o}{H}\right)} \tag{30}$$

where n_{no} refers to the neutral density at height r_o.

From equations (26), (28), (29) and (30) we get the following differential equation

$$\left(1 + \frac{\tan^2\theta}{4}\right) \frac{\partial^2 n_e}{\partial r^2} + \frac{\partial n_e}{\partial r} \left[\frac{3}{2H} + \frac{3}{r} + \frac{\tan^2\theta}{4} \left(\frac{3}{r} + \frac{1}{H}\right)\right]$$

$$+ \frac{n_e}{2H} \left[\left(\frac{1}{H} + \frac{3}{r}\right) + \frac{\tan^2\theta}{4} \left(\frac{3}{r} + \frac{1}{2H}\right)\right] = 0 \tag{31}$$

In deriving (31), the dipole field approximation, i.e., $\tan I = 2 \cot\theta$, has been assumed. The solution of equation (31) is given by

$$n_e(r,\theta) = A_1(\theta) e^{-\frac{r}{2H}} + A_2(\theta) e^{-\frac{r}{2H}} \int \frac{1}{r^3} e^{-\frac{2r}{H}} (4 + \tan^2\theta)\, dr \tag{32}$$

where $A_1(\theta)$ and $A_2(\theta)$ are two arbitrary functions of θ but are independent of r and must be evaluated to determine equation (32) uniquely. It is seen, however, that if equation (32) is substituted in equation (28) the only permissible values of $A_1(\theta)$ and $A_2(\theta)$ are when

$$A_1(\theta) = \text{const} = A_1$$

$$A_2(\theta) = 0$$

If we allow any other value of $A_2(\theta)$, the resulting differential equation of $A_2(\theta)$ has a solution which is not independent of r. This, however, is self-contradictory. We thus obtain

$$n_e = A_1 e^{-r/2H} \tag{33}$$

A similar result may be obtained if equation (31) is written in terms of variable of θ instead of variable of r. In this case we obtain the following differential equation.

$$\frac{\partial^2 n_e}{\partial \theta^2} + \left[\frac{r}{H} - (1+4\cot^2\theta)\right] \frac{\cot\theta}{1+4\cot^2\theta} \frac{\partial n_e}{\partial \theta} = 0 \tag{34}$$

The solution of equation (34) may be written in the form

$$n_e(r,\theta) = B_1(r) + B_2(r) \int \left(\frac{1+3\cos^2\theta}{4}\right)^{r/6H} \sin\theta \, d\theta \tag{35}$$

where $B_1(r)$ and $B_2(r)$ are arbitrary functions of r and must be evaluated from boundary conditions. Again substituting equation (35) in equation (28) we find, following the arguments given before, that

$$B_2(r) = 0$$

$$N_e = B_1(r) = \text{const } e^{-r/2H} \tag{36}$$

which is equivalent to equation (33).

Equation (36) corresponds to the hydrostatic distribution of electron density with a scale height twice that of the neutral and is also obtained when $\vec{v} = 0$.

As a result of the previous analysis it is clear that the assumption of ambipolar diffusion in the ionosphere requires that $\nabla\times \vec{v} \times \vec{B} = 0$. In studying the effect of ambipolar diffusion on the charged particle distribution in the ionosphere, this point must be taken into account. In particular, the assumption of plasma diffusion along the field lines leads to the condition of hydrostatic distribution implying that $\vec{v} = 0$, independent of geomagnetic latitude.

Discussion

The results obtained in the preceding section are not in agreement with the findings of Kendall (1962) Lyon (1963), Rishbeth et al (1963) and Goldberg and Schmerling (1963) even though all these authors have studied the case of plasma diffusion along the field lines. The disagreement arises due to the fact that they have not taken into account the additional constraint on ∇n_e given by equation (28) which automatically results from the assumption of field aligned plasma diffusion. However, in the opinion of this author this point must be taken into account. In view of the great geophysical importance of this problem we shall pursue this point a little further and investigate the limitations of solving this problem when $\vec{v}_e \neq \vec{v}_i$. For the conditions existing in the F-region where the gyro-frequency is much greater than the collision frequencies it is shown in the appendix that

$$\vec{v}_e = e(-\gamma_e+\delta_e\gamma_i)(\vec{E}.\vec{h})\vec{h} - (\gamma_e+\delta_e\gamma_i)(\frac{1}{n_e}\nabla p_e.\vec{h})\vec{h}$$

$$+ (m_e\gamma_e+m_i\delta_e\gamma_i)(\vec{g}.\vec{h})\vec{h} \qquad (37)$$

$$\vec{v}_i = e(\gamma_i - \delta_i\gamma_e)(E.\vec{h})\vec{h} - (\gamma_i + \delta_i\gamma_e)(\frac{1}{n_e}\nabla p_e.\vec{h})\vec{h}$$

$$+ (m_i\gamma_i + m_e\delta_i\gamma_e)(\vec{g}.\vec{h})\vec{h} \qquad (38)$$

where $\vec{h}$ is the unit vector along the field lines and the coefficient γ_e, γ_i etc. are defined by equation A-3 in the appendix. It is evident from equations (37) and (38) that both electron and ion velocities are along the magnetic field when their gyro frequencies are much greater than their respective collision frequencies. However, it is not at all evident that the magnitudes of the two velocities are equal. It is not possible to make any further simplification of equations (37) and (38) since the mathematical problem leading to the solution for the electric field is extremely difficult. In order to avoid this difficulty one generally assumes $\vec{v}_e = \vec{v}_i = \vec{v}$. In this case the electric field can be easily eliminated from equations (37) and (38) leading to the well known expression for the diffusion velocity.

$$\vec{v} = \frac{\gamma_e\gamma_i}{\gamma_e + \gamma_i}\left\{\left[\frac{2}{n_e}\nabla p_e + (m_e + m_i)\vec{g}\right].\vec{h}\right\}\vec{h} \qquad (39)$$

or in the component from

$$v_{er} = -D\left\{(\frac{1}{n_e}\frac{\partial n_e}{\partial r} + \frac{1}{T_e}\frac{\partial T_e}{\partial r} + \frac{1}{2H_i})\sin^2 I\right.$$

$$\left. + (\frac{1}{r}\frac{1}{n_e}\frac{\partial n_e}{\partial \theta} + \frac{1}{T_e}\frac{\partial T_e}{\partial \theta})\cos I \sin I\right\} \qquad (40)$$

$$v_{e\theta} = -D\left\{(\frac{1}{n_e}\frac{\partial n_e}{\partial r} + \frac{1}{T_e}\frac{\partial T_e}{\partial r} + \frac{1}{2H_i})\sin I \cos I\right.$$

$$\left. + (\frac{1}{r}\frac{1}{n_e}\frac{\partial n_e}{\partial \theta} + \frac{1}{T_e}\frac{\partial T_e}{\partial \theta})\cos^2 I\right\}$$

where $D = 2kT_e\frac{\gamma_e\gamma_i}{\gamma_e + \gamma_i}$ may be interpreted as diffusion coefficient.

Equation (40) has been the basis of studying the diffusion problem in the ionosphere. However, in the light of the discussion presented in this paper it is evident that under the assumption of $\vec{v}_e = \vec{v}_i$, the correct expression for $\vec{v}$ is given by equation (24) which does not explicitely depend on the magnetic field irrespective of the relative magnitude of gyro and collision frequencies. The dependence on the magnetic field comes from equation (22) which is the required condition for $\vec{v}_e = \vec{v}_i$.

Unfortunately, there is no simple way of solving for $\vec{v}_e$ and $\vec{v}_i$ in terms of the particle densities and temperatures. In order to avoid the mathematical complexities, it may perhaps be desirable to devise experimental techniques for measuring the electric field. This will certainly be an important step in our understanding of the very intricate problem of diffusion in the ionosphere.

Appendix

If $\vec{v}_n \ll \vec{v}_e$ or $\vec{v}_i$, we may rewrite equations 8 and 9 in the following form.

$$\vec{v}_e + \lambda_e \vec{v}_e \times \vec{h} = \gamma_e(-e\vec{E} + \frac{\vec{A}_e}{n_e}) + \delta_e \vec{v}_i \tag{A-1}$$

$$\vec{v}_i + \lambda_i \vec{v}_i \times \vec{h} = \gamma_i(e\vec{E} + \frac{\vec{A}_i}{n_i}) + \delta_i \vec{v}_e \tag{A-2}$$

where $\vec{h}$ is a unit vector along the field lines and the coefficients γ_e, γ_i etc. are given by the following equations.

$$\gamma_e = \frac{1}{n_i \alpha_{ei} + n_n \alpha_{en}}$$

$$\gamma_i = \frac{1}{n_e \alpha_{ei} + n_n \alpha_{in}}$$

$$\lambda_e = eB\gamma_e \tag{A-3}$$

$$\lambda_i = -\ eB\gamma_i$$

$$\delta_e = n_i\alpha_{ei}\gamma_e$$

$$\delta_i = n_e\alpha_{ei}\gamma_i$$

The coefficient λ_e and λ_i may be interpreted as the ratio of gyrofrequency and the effective collision frequencies of electrons and ions respectively. Further, δ_e and δ_i are the coupling terms between electron and ion motion through collision. Obviously, δ_e and δ_i are zero if $\vec{v}_e = \vec{v}_i$ since the terms containing α_{ei} are zero.

Equations (A-1) and (A-2) may be solved for $\vec{v}_e$ and $\vec{v}_i$. We thus obtain

$$\vec{v}_e = \frac{1-\lambda_e\lambda_i-\delta_e\delta_i}{(1-\lambda_e\lambda_i-\delta_e\delta_i)^2+(\lambda_e+\lambda_i)^2}\left\{\gamma_e\vec{G}_e\left[1+\frac{\lambda_i(\lambda_e+\lambda_i)}{1-\lambda_e\lambda_i-\delta_e\delta_i}\right]\right.$$

$$+\frac{\gamma_e(\vec{G}_e\cdot\vec{h})\vec{h}}{1-\delta_e\delta_i}\left[\frac{(\lambda_e+\lambda_i)(\lambda_e+\lambda_i\delta_e\delta_i)}{1-\lambda_e\lambda_i-\delta_e\delta_i}-\lambda_i\lambda_e\right]$$

$$+\gamma_e\vec{h}\times\vec{G}_e\left[\frac{\lambda_e+\lambda_i}{1-\lambda_e\lambda_i-\delta_e\delta_i}-\lambda_i\right]$$

$$+\delta_e\gamma_i\vec{G}_i+\frac{\delta_e\gamma_i(\vec{G}_i\cdot\vec{h})\vec{h}}{1-\delta_e\delta_i}\left[\frac{(\lambda_e+\lambda_i)^2}{1-\lambda_e\lambda_i-\delta_e\delta_i}-\lambda_i\lambda_e\right]$$

$$\left.+\delta_e\gamma_i\,\vec{h}\times\vec{G}_i\left[\frac{\lambda_e+\lambda_i}{1-\lambda_e\lambda_i-\delta_e\delta_i}\right]\right\} \qquad \text{(A-4)}$$

where

$$\vec{G}_e = -\,e\vec{E} + \frac{\vec{A}_e}{n_e}$$

$$\vec{G}_i = e\,\vec{E} + \frac{\vec{A}_i}{n_i} \qquad \text{(A-5)}$$

A similar expression can be written for $\vec{v}_i$ by interchanging the suffixes e and i.

At this stage it is appropriate to make a numerical estimate of the coefficients λ_e, γ_e, etc. in order to get a physical insight of equation (A-4). For the conditions existing in the F-region, we adopt the following numerical values of the collision frequencies as given by Chapman (1956). These frequencies correspond to his model h(T = 1480°). Thus we may write

$$\nu_{ei} = 268/\text{sec}$$

$$\nu_{en} = 37.4/\text{sec}$$

$$\nu_{in} = 1/\text{sec}$$

Again $\omega_e = \frac{eB}{m_e} = 5.2 \times 10^6/\text{sec}$ where $B = 0.3 \times 10^{-4}$ Weber

$\omega_i = \frac{eB}{m_i} = 1.5 \times 10^2/\text{sec}$ assuming $m_i = 19$ amu

Substituting these values in equation (39) and neglecting m_e as compared to m_i we obtain

$$\lambda_e \simeq 1.7 \times 10^4$$

$$\lambda_i \simeq 2.8 \times 10^2$$

$$\gamma_i \simeq 0.6 \times 10^{26} \text{ sec/Kgm}$$

$$\gamma_e \simeq 3.5 \times 10^{27} \text{ sec/Kgm}$$

$$\delta_e \simeq .88$$

$$\delta_i \simeq 1.54 \times 10^{-2}$$

From the above computations it is clear that λ_e and $\lambda_i >> 1$; $\delta_e \simeq 1$ and $\delta_i << 1$. With these approximations, equation (A-4) can be written in the following from

$$\vec{v}_e = \gamma_e \left[\frac{\vec{G}_e}{\lambda_e^2} + (\vec{G}_e . \vec{h})\vec{h} + \frac{\vec{h} \times \vec{G}_e}{\lambda_e}\right] + \delta_e \gamma_i \left[- \frac{\vec{G}_i}{\lambda_e \lambda_i}\right.$$

$$\left. + (\vec{G}_i . \vec{h})\vec{h} + \frac{\vec{h} \times \vec{G}_i}{\lambda_e \lambda_i} \left(\frac{1}{\lambda_i} + \frac{1}{\lambda_e}\right)\right] \qquad \text{(A-6)}$$

A further approximation of equation (A-6) results in the following equation.

$$\vec{v}_e \simeq \gamma_e (\vec{G}_e . \vec{h})\vec{h} + \delta_e \gamma_i (\vec{G}_i . \vec{h})\vec{h} \qquad \text{(A-7)}$$

Following the same procedure for $\vec{v}_i$, we may write

$$\vec{v}_i \simeq \gamma_i (\vec{G}_i . \vec{h})\vec{h} + \delta_i \gamma_e (\vec{G}_e . \vec{h})\vec{h} \qquad \text{(A-8)}$$

We may further write, assuming $\frac{1}{n_e} \nabla p_e \simeq \frac{1}{n_i} \nabla p_i$, and substituting for $\vec{G}_e$ and $\vec{G}_i$ from equation (A-5)

$$\vec{v}_e = e(-\gamma_e + \delta_e \gamma_i)(\vec{E} . \vec{h})\vec{h} - (\gamma_e + \delta_e \gamma_i)\left(\frac{1}{n_e} \nabla p_e . \vec{h}\right)\vec{h}$$

$$+ (m_e \gamma_e + m_i \delta_e \gamma_i)(\vec{g} . \vec{h})\vec{h} \qquad \text{(A-9)}$$

$$\vec{v}_i = e(\gamma_i - \delta_i \gamma_e)(E.\vec{h})\vec{h} - (\gamma_i + \delta_i \gamma_e)(\frac{1}{n_e} \nabla p_e . \vec{h})\vec{h}$$

$$+ (m_i \gamma_i + m_e \delta_i \gamma_e)(\vec{g}.\vec{h})\vec{h} \qquad \text{(A-10)}$$

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